NONLINEAR IMAGE INTERPOLATION THROUGH EXTENDED PERMUTATION FILTERS

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ABSTRACT

This paper examines the application of *extended permutation rank selection* (EPRS) filters in image interpolation. EPRS filters are constrained to output an order statistic based on N observation samples and K statistics that are functions of the observation samples. By including the sample mean as the sole statistic when K=1, EPRS filters were shown to have superior edge-enhancing properties. By letting K=0, we can define a subset class known as *rank conditioned rank selection* (RCRS) filters. We will show that these filters can also be applied to the issue of image interpolation. In this case, by extending the observation space by including more original samples and by adding additional linear statistics, EPRS filters produce results superior to traditional methods in the application of image interpolation.

1. INTRODUCTION

Image interpolation is an issue that has recently received great attention. With the continued development of data communication schemes for image transmission over the Internet and image/video coding for HDTV, novel techniques for image down-sampling and image interpolation are more sought after than ever before. Down-sampling schemes are already well developed, and it is the interpolation phase that provides the most room for development of new methods.

Image interpolation is typically performed by creating an observation vector consisting of the samples in the immediate neighborhood of the original pixel to be interpolated. Applying a traditional linear filter to the observation vector exploits the spatial ordering of the neighboring pixels by outputting a weighted sum combination. This method tends to produce blurred edges and smoothed details. Nonlinear filters, however, have proven more successful in this area. Zeng examined a series of median filters, in which the observation set includes the original four neighboring samples [1]. He further investigated extending the observation vector to include mean statistics based on the four samples, generally seeing improved results.

Median filtering itself can, however, eliminate fine details such as sharp corners and narrow lines because the rank-based filtering process neglects any spatial neighborhood information. We propose to use *rank conditioned rank selection* (RCRS) and *extended permutation rank selection* (EPRS) filters, which incorporate both the rank and spatial ordering of the observation samples [2],[3]. RCRS filters are a class of filters based upon the partitioning of an observation space using rank permutations. A rank selection filtering operation is defined over each individual partition, allowing the RCRS filter to output one of the original samples from the filter window. The observation vector used to generate the permutations consist of the original N observation samples. EPRS filters use an extended observation vector that also includes K statistics of the N observation samples. This allows the rank selection operation of the EPRS filter to output not only one of the N original samples, but one of the K statistics as well.

By selecting the appropriate N original samples and K statistics, both RCRS and EPRS filters can be applied to image interpolation. This paper will concentrate on the interpolation of quinqunx sub-sampled images. We selected this sub-sampling lattice as our focus because it warrants a single interpolator structure as defined in the next section. With the judicious selection of the original samples and corresponding statistics, both RCRS and EPRS filters provide superior interpolation results compared to traditional methods.

This remainder of this paper is organized as follows. In Section 2, the quincunx sub-sampling lattice is presented. Various nonlinear interpolators for the lattice are defined in Section 3. In Section 4, we present experimental results.

2. QUINQUNX SUB-SAMPLING LATTICE

The initial stage of the quinqunx sub-sampling/interpolation progression can be seen in Fig. 1. The quinqunx lattice reduces the number of samples by 2 [1],[4]. The lattice can subsequently be zero-interlaced to produce the original size image (Fig. 1-c) [5]. The next step in the interpolation pro-

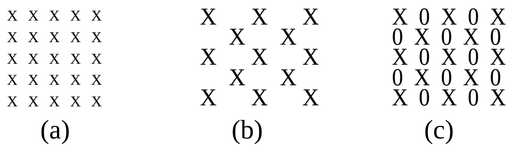


Fig. 1. Sampling lattices. (a)Original. (b)Quinqunx subsampled. (c)Quinqunx zero-interlaced.

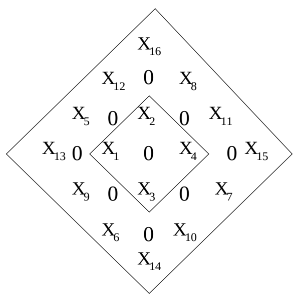


Fig. 2. Interpolator Structure for Quinquinx Zero Interlace.

gression is to filter the zero pixels in the zero-interlaced image, producing the reconstructed image. One preliminary point to consider is whether the original image should be low-pass filtered before it is subsampled to avoid any aliasing. We decided against this option since the low-pass filter would remove the high frequencies and sharp quality that we are trying to preserve through nonlinear interpolation. This also represents the case common in many practical applications.

The basic interpolator structure for the quinqunx zero interlace is shown in Fig. 2. The central zero is pixel to be filtered, or interpolated. The inner diamond encompasses the four adjacent neighboring samples, while the outer diamond encompasses the next set of known samples. Thus, using the given interpolator structure, there are up to 16 known samples that we can use to interpolate the value for the central zero.

3. QUINQUNX INTERPOLATORS

Consider the 2–dimensional discrete sequences $\{d(\mathbf{n})\}$ and $\{x(\mathbf{n})\}$,where the discrete index $\mathbf{n} = [n_1, n_2]$. Let these sequences represent the original image (Fig. 1-a) and the zero-interlaced version (Fig. 1-c) respectively. Note that from this point on, the index $\mathbf{n}$ is assumed and is used explicitly only when needed. Also, consider a 2–dimensional window function that spans N samples and passes over the zero-interlaced image, only filtering the appropriate zero pixels. At a given zero pixel location, we can consider the surrounding non-zero samples to create an observation window of N samples, defining a corresponding observation vector of

$$\mathbf{x}_N = [x_1, x_2, \ldots, x_N]. \tag{1}$$

For example, using the interpolator structure from Fig. 2, we can extract the observation vector $\mathbf{x}_{16} = [x_1, x_2, \ldots, x_{16}]$ to create a 16 element filter window.

We can pass the observation vector $\mathbf{x}_N$ through any filter, using the output as the value for the center zero pixel. A typical linear scheme would exploit the spatial ordering of the observed samples by outputting a weighted average of $\mathbf{x}_N$. Increasing the computation and complexity, we could exploit the rank ordering of the samples by choosing the median of $\mathbf{x}_N$. We, however, propose a scheme combining both the rank and spatial ordering of the samples.

Additional information about the window can be extracted by extending the observation vector to include K characteristic statistics. Thus, we define an extended observation vector as

$$\begin{aligned} \tilde{\mathbf{x}}_{N,K} &= [\tilde{x}_1, \tilde{x}_2, \ldots, \tilde{x}_{N+K}], \qquad (2)\\ &= [x_1, x_2, \ldots, x_N, F_1(\mathbf{x}_N), \ldots, F_K(\mathbf{x}_N)]. \end{aligned}$$

where $F_i(\mathbf{x}_N)$ is some function of the original observation vector. Note that the standard observation vector $\mathbf{x}_N$ is simply a special case of $\tilde{\mathbf{x}}_{N,K}$, for $K = 0$. Hence the RCRS filter, which is based on the original observation vector, is a limiting case of the EPRS filter that uses an extended observation vector of $\tilde{\mathbf{x}}_{N,0}$. For the remainder of this section, we will only explicitly define the EPRS filter.

To create a rank ordering of the extended observation vector, sort $\tilde{\mathbf{x}}_{N,K}$ by rank to define

$$\tilde{x}_{(1)} \leq \tilde{x}_{(2)} \leq \cdots \leq \tilde{x}_{(N+K)}. \tag{3}$$

The ranks of the N samples and K statistics can be related to their spatial position (within the observation vector) by defining r_i to be the rank of the sample at index i of $\tilde{\mathbf{x}}_{N,K}$. This allows for the creation of a rank vector $\mathbf{r} = [r_1, r_2, \ldots, r_{N+K}]$, which contains the ranks of the N samples and K statistics. This rank vector can be used as an input into a selection rule $\mathcal{S}(\cdot)$, which will produce desired rank to output [2]. This allows the center zero pixel to take on any value in $\tilde{\mathbf{x}}_{N,K}$.

Consider a subset of the full rank vector where we only rank M original samples and L statistics, such that $0 \leq M \leq N$ and $0 \leq L \leq K$. We can now define the rank feature vector to be

$$\mathbf{r}* = [r_{\gamma_1}, r_{\gamma_2}, \ldots, r_{\gamma_M}, r_{\beta_1}, r_{\beta_2}, \ldots, r_{\beta_L}], \tag{4}$$

where $1 \leq \gamma_i \leq N$, $N+1 \leq \beta_i \leq N+K$. We will refer to M as the order of the EPRS filter.

Finally, we address the issue of the number of elements in the extended observation vector. Median filters, when given an even-numbered $\tilde{\mathbf{x}}_{N,K}$, output the average of the two center-ranked samples. This behavior proves to be very desirable in many cases. To allow EPRS filters the ability to do the same, define

$$\hat{\mathbf{x}} = \begin{cases} \tilde{\mathbf{x}}_{N,K} & \text{if } N+K \text{ is odd} \\ [\tilde{\mathbf{x}}_{N,K}, \text{median}(\tilde{\mathbf{x}}_{N,K})] & \text{if } N+K \text{ is even} \end{cases} \quad (5)$$

The vector $\hat{\mathbf{x}}$ is identical to $\tilde{\mathbf{x}}_{N,K}$, except a median value is appended when $\tilde{\mathbf{x}}_{N,K}$ has an even number of elements. Defining $\hat{x}_{(i)}$ as the i^{th} rank-ordered element of $\hat{\mathbf{x}}$, the output of filter can now be defined as

$$F_{EPRS}(\mathbf{x}) = \hat{x}_{(\mathcal{S}(\mathbf{r}*))}, \quad (6)$$

The EPRS selection filter rule $\mathcal{S}(\cdot)$ can be optimized under the least mean absolute error (MAE) or mean squared error (MSE) given the training sequences $\{d(\mathbf{n})\}$ and $\{x(\mathbf{n})\}$. The procedure is described in [2].

To examine the effects of N and K on the performance of the EPRS filters, define two sets of observation vectors. In the initial set, consider the original observation vector for K = 0 (corresponding to a RCRS filter). Define 9 observation vectors for N = 2,3,...,8,12,16 as

$$\mathbf{x}_i = [x_1, x_2, \ldots, x_i], \quad (7)$$

where $i = 2, 3, \ldots, 8, 12, 16$.

To examine the effect of extending the observation vector, define a set of extended observation vectors based on the original vectors above. Noting the success of adding an overall mean to EPRS filters as an edge enhancement mechanism [3], we will also extend our filters with linear averaging statistics as well. However, to derive increased statistical information about our observation window, we will add directional means of samples positioned along lines which pass through the center sample.

$$\tilde{\mathbf{x}}_{4,2} = [\mathbf{x}_4, \bar{x}_{\{1,4\}}, \bar{x}_{\{2,3\}}]. \quad (8)$$

$$\tilde{\mathbf{x}}_{8,2} = [\mathbf{x}_8, \bar{x}_{\{1,2,3,4,5,7\}}, \bar{x}_{\{1,2,3,4,6,8\}}]. \quad (9)$$

$$\tilde{\mathbf{x}}_{12,2} = [\mathbf{x}_{12}, \bar{x}_{\{1,2,3,4,5,7,10,12\}}, \bar{x}_{\{1,2,3,4,6,8,9,11\}}] \quad (10)$$

$$\tilde{\mathbf{x}}_{16,4} = [\mathbf{x}_{16}, \bar{x}_{\{1,2,3,4,5,7,10,12\}}, \bar{x}_{\{1,2,3,4,6,8,9,11\}}, \bar{x}_{\{2,3,14,16\}}, \bar{x}_{\{1,4,13,15\}}]. \quad (11)$$

where $\bar{x}_{\{i1,i2,\ldots,in\}}$ = mean$(x_{i1}, x_{i2}, \ldots, x_{in})$. Note that because of the added symmetry in $\mathbf{x}_{16}$, we were able to directly extend the vector by four averaging statistics denoting the four main directional lines passing through the center pixel.

All of the defined observation vectors and extended observation vectors can be used as input vectors for linear and median interpolation schemes as well as the EPRS method. In the linear and median schemes, this results in simply taking a weighted sum or the median of $\tilde{\mathbf{x}}_N$ and $\tilde{\mathbf{x}}_{N,K}$, while for EPRS scheme, the selection rule $\mathcal{S}$ is used to output one of the samples in the vector.

4. RESULTS

Experimental results are presented for the test image of Cafe. This grayscale image is 2560 by 2048 pixels. A large image size was needed to successfully train and optimize the selection rule $\mathcal{S}$. The simulations consisted of subsampling cafe using the quinqunx lattice without pre-filtering. We interpolated the sub-sampled image back to its original size using the various EPRS filters defined above. We also compare the results with linear, median, and *weighted median* (WM) [5] filters. The weights for the linear filter were the inverse of the euclidean distances of each sample to the center pixel. The weights for the WM filter were the coefficents of a linear Weiner filters optimized on Cafe.

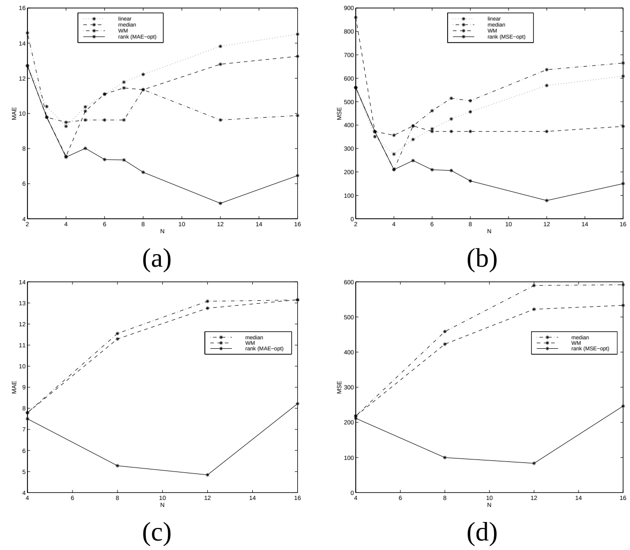
The median, WM, and EPRS filters follow a progression based on both the orginal and extended observation vectors. The linear filter, however, was not applied to the extended vectors since the appended statitics were are linear and yeiled no performance gain. Analogously, since no Weiner coefficents for the extended observation vectors could be derived, the weights for the WM filter for those vectors were simply the inverse of the euclidean distance of the given sample to the center. The weights for the additional statitics were equal to the mean distance of the averaged samples to the center

Figure 4-a and Figure 4-b gives the MAE and the MSE between the original and the reconstructed image using the original observation vectors (K = 0). Two EPRS filters are presented for each observation vector; one that was optimized under the MAE criterion (Fig. 4-a), and one that was optimized under the MSE criterion (Fig. 4-b). For window sizes $N = 1, 2, \ldots, 8$, all the samples in window are used in the rank vector (i.e., $M = N$). For $N = 12$, only the first 6 samples $x_1, x_2, \ldots, x_6$ from $\mathbf{x}_{12}$ are used. And for $N = 16$, we only rank the first 4 samples from $\mathbf{x}_{16}$.For the extended vectors (Fig. 4-c and Fig. 4-d), we always ranked all the K statistics ($L = K$ for all the vectors). However, we only ranked the first 4 original samples of $\mathbf{x}_{4,2}, \mathbf{x}_{8,2}$, and $\mathbf{x}_{12,2}$. Due to memory restrictions, we were only able to rank the first original sample of $\mathbf{x}_{16,4}$.

In all cases, the linear and median schemes' performance peak for $\mathbf{x}_4$. However, as N is increased beyond 4, rank interpolation is the only scheme that performs better. For $N = 16$, the rank interpolation schemes seem to worsen, but this can be attributed to the low filter order of $M = 1$. We hypothesize that with an adequate filter order, the rank filtering would have been able to handle the larger window size.

Sample figures are presented in Fig. 5. Here, we apply an EPRS filter trained on Cafe (Fig. 3) to Aerial (Fig. 5-a). We consider the difference image between the original and reconstructed Aerial images. The results for the linear, WM, and MAE-optimized rank interpolation for the $\mathbf{x}_{12}$ ob-

Fig. 3. Training image of Cafe

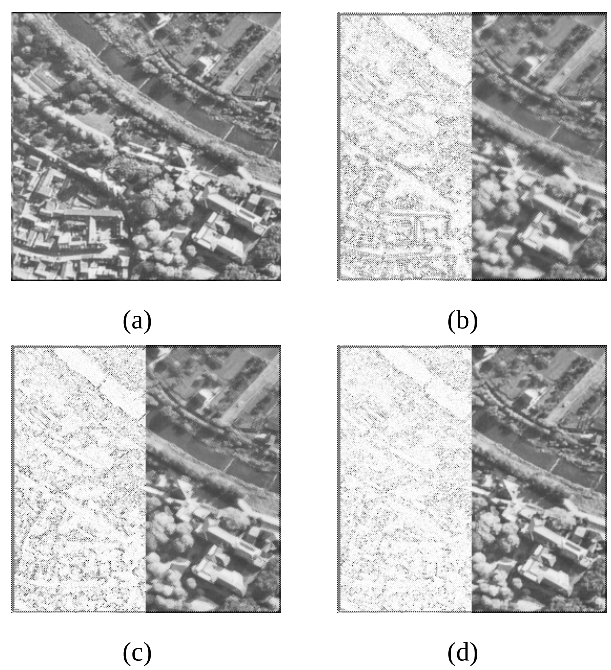


(a) (b) (c) (d)

Fig. 4. Summary of results (a)MAE and (b)MSE results for original observation vectors. (c)MAE and (d)MSE results for extended observation vectors.

servation vector are presented. The order used for the rank filter was 4. We see that the majority of error in both the linear and WM interpolation schemes stem from the edges within the image, which suggests that the rank interpolation successfully interpolates sharper edges then either method.

Generally, we can conclude that the rank ordering information is indeed useful and generically applicable. Furthermore, as the amount of rank-ordered information is increased (the further $\tilde{\mathbf{x}}_{N,K}$ is enlarged by increasing N or extended by increasing K), the better the performance of the rank filters.



(a) (b) (c) (d)

Fig. 5. Interpolation of Aerial based on the observation vector $\mathbf{x}_{12}$ using training data from Cafe. (a)Original. (b)Linear. (c)WM. (d) EPRS with $M = 4$.

5. REFERENCES

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